10.5.4 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 10 materials.

For help with Exercises 1 - 24, click one or more of the resources below:

- The graphs of the six trigonometric functions
- Transformations of the graphs of the six trigonometric functions

In Exercises 1 - 12, graph one cycle of the given function. State the period, amplitude, phase shift and vertical shift of the function.

1. $y = 3\sin(x)$
2. $y = \sin(3x)$
3. $y = -2\cos(x)$
4. $y = \cos\left(x - \frac{\pi}{2}\right)$
5. $y = -\sin\left(x + \frac{\pi}{3}\right)$
6. $y = \sin(2x - \pi)$
7. $y = -\frac{1}{3}\cos\left(\frac{1}{2}x + \frac{\pi}{3}\right)$
8. $y = \cos(3x - 2\pi) + 4$
9. $y = \sin\left(-x - \frac{\pi}{4}\right) - 2$
10. $y = \frac{2}{3}\cos\left(\frac{\pi}{2} - 4x\right) + 1$
11. $y = -\frac{3}{2}\cos\left(2x + \frac{\pi}{3}\right) - \frac{1}{2}$
12. $y = 4\sin(-2\pi x + \pi)$

In Exercises 13 - 24, graph one cycle of the given function. State the period of the function.

13. $y = \tan\left(x - \frac{\pi}{3}\right)$
14. $y = 2\tan\left(\frac{1}{4}x\right) - 3$
15. $y = \frac{1}{3}\tan(-2x - \pi) + 1$
16. $y = \sec\left(x - \frac{\pi}{2}\right)$
17. $y = -\csc\left(x + \frac{\pi}{3}\right)$
18. $y = -\frac{1}{3}\sec\left(\frac{1}{2}x + \frac{\pi}{3}\right)$
19. $y = \csc(2x - \pi)$
20. $y = \sec(3x - 2\pi) + 4$
21. $y = \csc\left(-x - \frac{\pi}{4}\right) - 2$
22. $y = \cot\left(x + \frac{\pi}{6}\right)$
23. $y = -11\cot\left(\frac{1}{5}x\right)$
24. $y = \frac{1}{3}\cot\left(2x + \frac{3\pi}{2}\right) + 1$

In Exercises 25 - 34, use Example 10.5.3 as a guide to show that the function is a sinusoid by rewriting it in the forms $C(x) = A\cos(\omega x + \phi) + B$ and $S(x) = A\sin(\omega x + \phi) + B$ for $\omega > 0$ and $0 \leq \phi < 2\pi$.

25. $f(x) = \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1$
26. $f(x) = 3\sqrt{3}\sin(3x) - 3\cos(3x)$
27. $f(x) = -\sin(x) + \cos(x) - 2$
28. $f(x) = -\frac{1}{2}\sin(2x) - \frac{\sqrt{3}}{2}\cos(2x)$

29. $f(x) = 2\sqrt{3}\cos(x) - 2\sin(x)$

30. $f(x) = \dfrac{3}{2}\cos(2x) - \dfrac{3\sqrt{3}}{2}\sin(2x) + 6$

31. $f(x) = -\dfrac{1}{2}\cos(5x) - \dfrac{\sqrt{3}}{2}\sin(5x)$

32. $f(x) = -6\sqrt{3}\cos(3x) - 6\sin(3x) - 3$

33. $f(x) = \dfrac{5\sqrt{2}}{2}\sin(x) - \dfrac{5\sqrt{2}}{2}\cos(x)$

34. $f(x) = 3\sin\left(\dfrac{x}{6}\right) - 3\sqrt{3}\cos\left(\dfrac{x}{6}\right)$

35. In Exercises 25 - 34, you should have noticed a relationship between the phases ϕ for the $S(x)$ and $C(x)$. Show that if $f(x) = A\sin(\omega x + \alpha) + B$, then $f(x) = A\cos(\omega x + \beta) + B$ where $\beta = \alpha - \dfrac{\pi}{2}$.

36. Let ϕ be an angle measured in radians and let $P(a, b)$ be a point on the terminal side of ϕ when it is drawn in standard position. Use Theorem 10.3 and the sum identity for sine in Theorem 10.15 to show that $f(x) = a\,\sin(\omega x) + b\,\cos(\omega x) + B$ (with $\omega > 0$) can be rewritten as $f(x) = \sqrt{a^2 + b^2}\sin(\omega x + \phi) + B$.

37. With the help of your classmates, express the domains of the functions in Examples 10.5.4 and 10.5.5 using extended interval notation. (We will revisit this in Section 10.7.)

In Exercises 38 - 43, verify the identity by graphing the right and left hand sides on a calculator.

38. $\sin^2(x) + \cos^2(x) = 1$

39. $\sec^2(x) - \tan^2(x) = 1$

40. $\cos(x) = \sin\left(\dfrac{\pi}{2} - x\right)$

41. $\tan(x + \pi) = \tan(x)$

42. $\sin(2x) = 2\sin(x)\cos(x)$

43. $\tan\left(\dfrac{x}{2}\right) = \dfrac{\sin(x)}{1 + \cos(x)}$

In Exercises 44 - 50, graph the function with the help of your calculator and discuss the given questions with your classmates.

44. $f(x) = \cos(3x) + \sin(x)$. Is this function periodic? If so, what is the period?

45. $f(x) = \frac{\sin(x)}{x}$. What appears to be the horizontal asymptote of the graph?

46. $f(x) = x\sin(x)$. Graph $y = \pm x$ on the same set of axes and describe the behavior of f.

47. $f(x) = \sin\left(\frac{1}{x}\right)$. What's happening as $x \to 0$?

48. $f(x) = x - \tan(x)$. Graph $y = x$ on the same set of axes and describe the behavior of f.

49. $f(x) = e^{-0.1x}\left(\cos(2x) + \sin(2x)\right)$. Graph $y = \pm e^{-0.1x}$ on the same set of axes and describe the behavior of f.

50. $f(x) = e^{-0.1x}\left(\cos(2x) + 2\sin(x)\right)$. Graph $y = \pm e^{-0.1x}$ on the same set of axes and describe the behavior of f.

51. Show that a constant function f is periodic by showing that $f(x+117) = f(x)$ for all real numbers x. Then show that f has no period by showing that you cannot find a *smallest* number p such that $f(x+p) = f(x)$ for all real numbers x. Said another way, show that $f(x+p) = f(x)$ for all real numbers x for ALL values of $p > 0$, so no smallest value exists to satisfy the definition of 'period'.

Checkpoint Quiz 10.5

1. Graph one cycle of $y = 3 - \cos\left(\dfrac{x-\pi}{2}\right)$. Find the phase shift, period, and amplitude.

2. Graph one cycle of $y = 2\csc(3x) + 1$. Find the period.

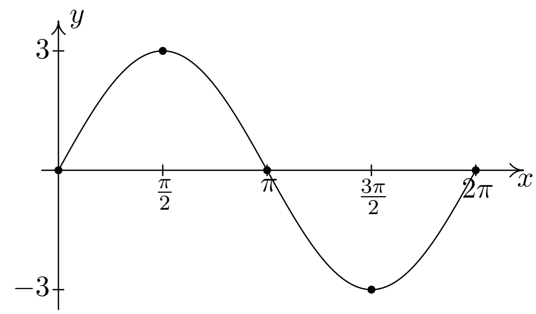
3. Graph one cycle of $y = \tan\left(\dfrac{\pi}{2} - x\right)$

For worked out solutions to this quiz, click the links below:

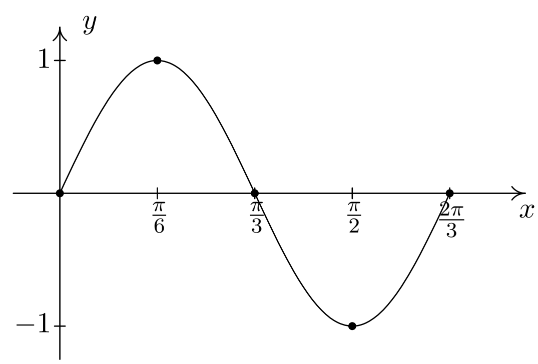
- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3
- Quiz Solution Part 4

10.5.5 Answers

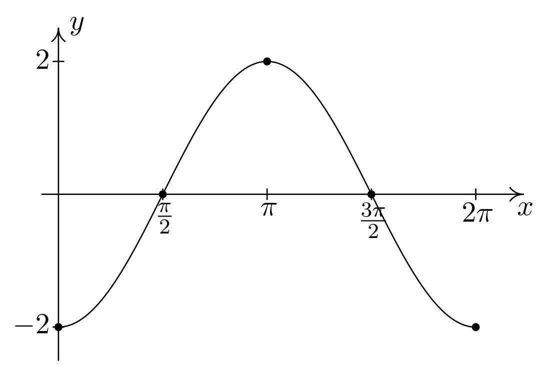
1. $y = 3\sin(x)$
   Period: 2π
   Amplitude: 3
   Phase Shift: 0
   Vertical Shift: 0



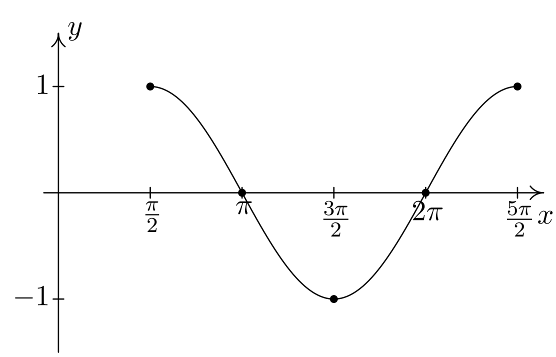
2. $y = \sin(3x)$
   Period: $\dfrac{2\pi}{3}$
   Amplitude: 1
   Phase Shift: 0
   Vertical Shift: 0



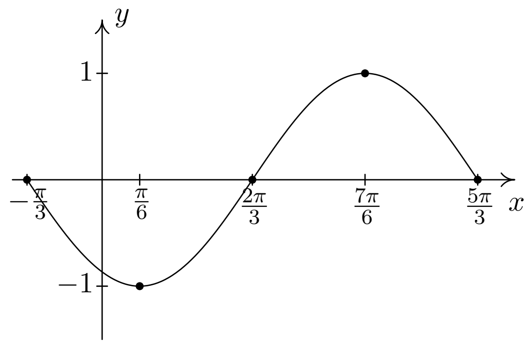
3. $y = -2\cos(x)$
   Period: 2π
   Amplitude: 2
   Phase Shift: 0
   Vertical Shift: 0



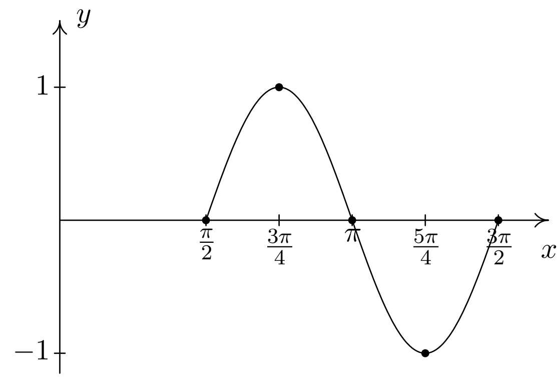
4. $y = \cos\left(x - \dfrac{\pi}{2}\right)$
   Period: 2π
   Amplitude: 1
   Phase Shift: $\dfrac{\pi}{2}$
   Vertical Shift: 0



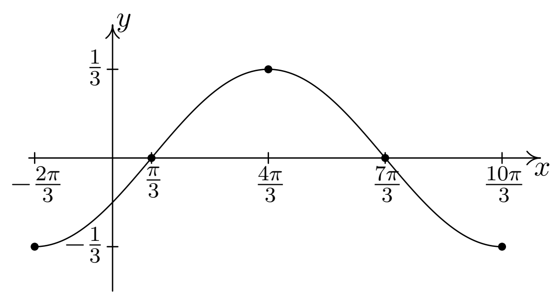
5. $y = -\sin\left(x + \dfrac{\pi}{3}\right)$
   Period: 2π
   Amplitude: 1
   Phase Shift: $-\dfrac{\pi}{3}$
   Vertical Shift: 0



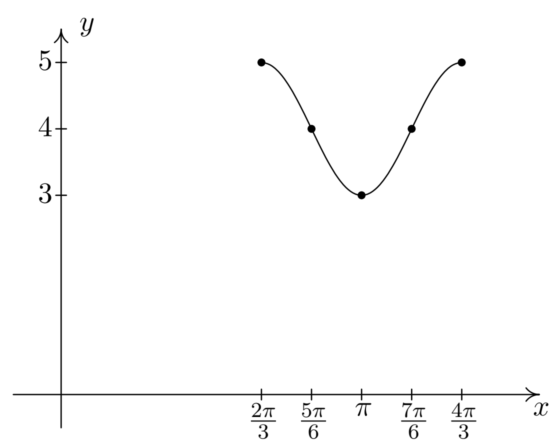
6. $y = \sin(2x - \pi)$
   Period: π
   Amplitude: 1
   Phase Shift: $\dfrac{\pi}{2}$
   Vertical Shift: 0



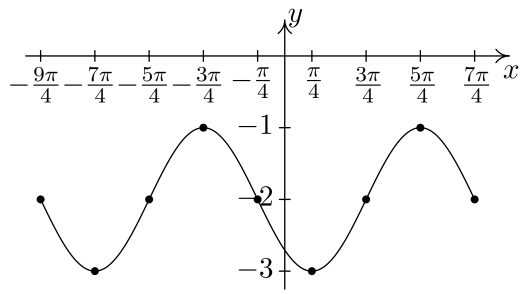
7. $y = -\dfrac{1}{3}\cos\left(\dfrac{1}{2}x + \dfrac{\pi}{3}\right)$
   Period: 4π
   Amplitude: $\dfrac{1}{3}$
   Phase Shift: $-\dfrac{2\pi}{3}$
   Vertical Shift: 0



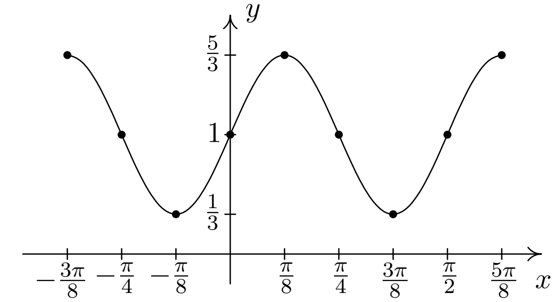
8. $y = \cos(3x - 2\pi) + 4$
   Period: $\dfrac{2\pi}{3}$
   Amplitude: 1
   Phase Shift: $\dfrac{2\pi}{3}$
   Vertical Shift: 4



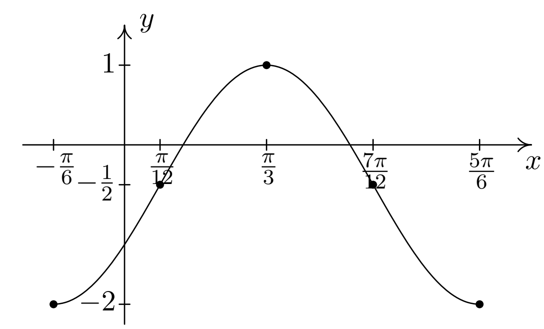
9. $y = \sin\left(-x - \dfrac{\pi}{4}\right) - 2$
   Period: 2π
   Amplitude: 1
   Phase Shift: $-\dfrac{\pi}{4}$ (You need to use $y = -\sin\left(x + \dfrac{\pi}{4}\right) - 2$ to find this.)[15]
   Vertical Shift: -2



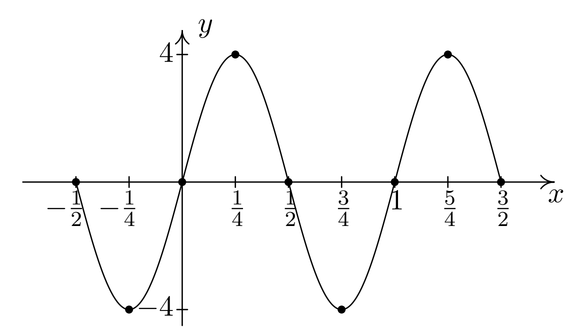
10. $y = \dfrac{2}{3}\cos\left(\dfrac{\pi}{2} - 4x\right) + 1$
    Period: $\dfrac{\pi}{2}$
    Amplitude: $\dfrac{2}{3}$
    Phase Shift: $\dfrac{\pi}{8}$ (You need to use $y = \dfrac{2}{3}\cos\left(4x - \dfrac{\pi}{2}\right) + 1$ to find this.)[16]
    Vertical Shift: 1



11. $y = -\dfrac{3}{2}\cos\left(2x + \dfrac{\pi}{3}\right) - \dfrac{1}{2}$
    Period: π
    Amplitude: $\dfrac{3}{2}$
    Phase Shift: $-\dfrac{\pi}{6}$
    Vertical Shift: $-\dfrac{1}{2}$

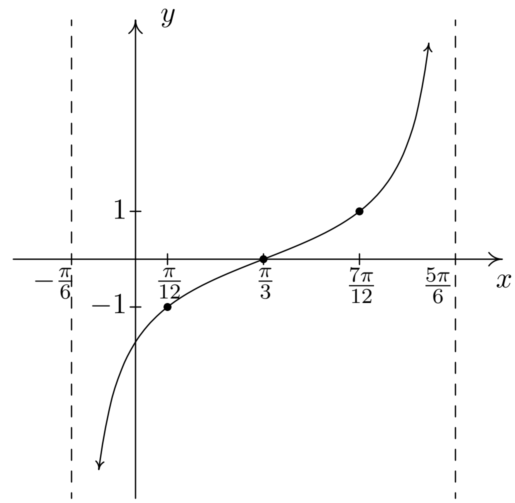


12. $y = 4\sin(-2\pi x + \pi)$
    Period: 1
    Amplitude: 4
    Phase Shift: $\dfrac{1}{2}$ (You need to use $y = -4\sin(2\pi x - \pi)$ to find this.)[17]
    Vertical Shift: 0

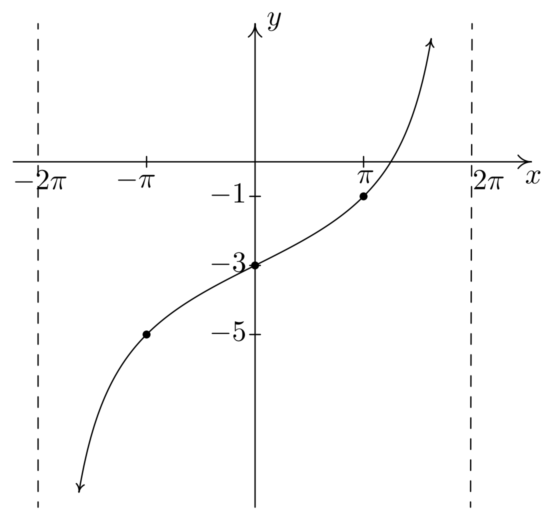


[15] Two cycles of the graph are shown to illustrate the discrepancy discussed on page 834.
[16] Again, we graph two cycles to illustrate the discrepancy discussed on page 834.
[17] This will be the last time we graph two cycles to illustrate the discrepancy discussed on page 834.

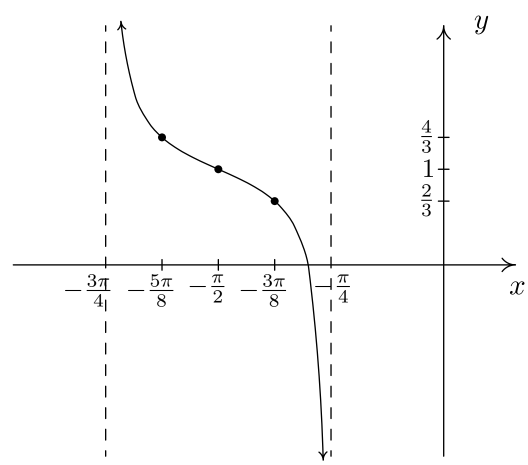
13. $y = \tan\left(x - \dfrac{\pi}{3}\right)$
Period: π



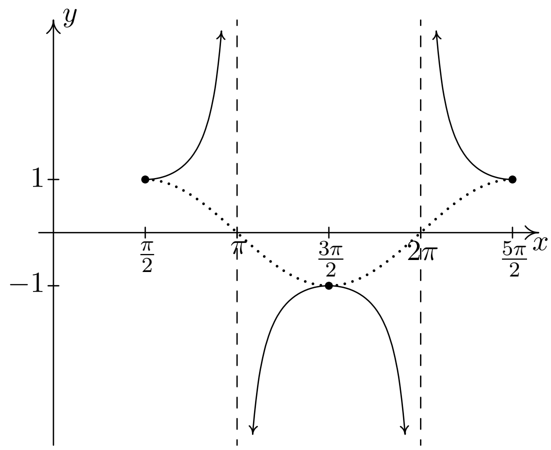
14. $y = 2\tan\left(\dfrac{1}{4}x\right) - 3$
Period: 4π



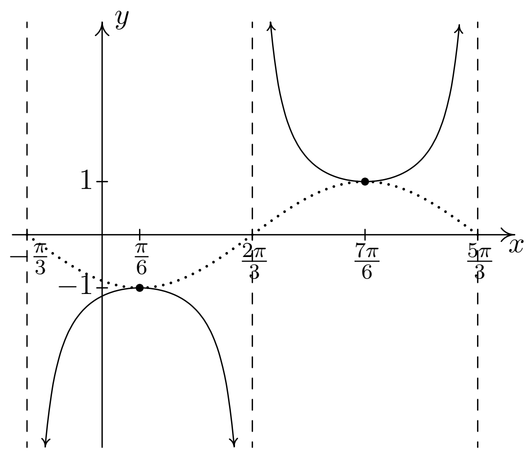
15. $y = \dfrac{1}{3}\tan(-2x - \pi) + 1$
is equivalent to
$y = -\dfrac{1}{3}\tan(2x + \pi) + 1$
via the Even / Odd identity for tangent.
Period: $\dfrac{\pi}{2}$



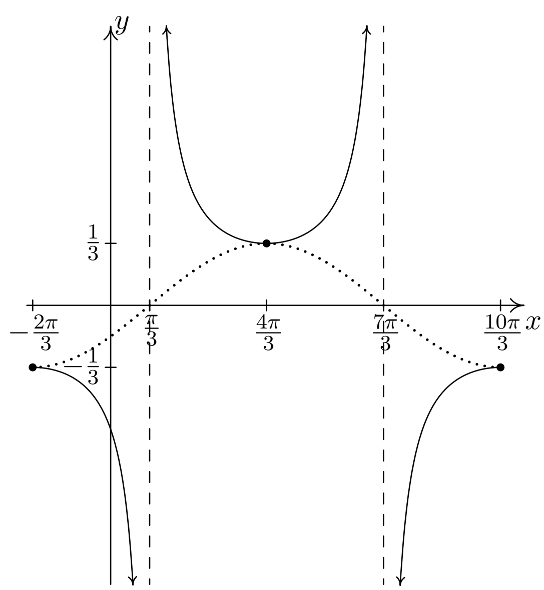
16. $y = \sec\left(x - \frac{\pi}{2}\right)$
Start with $y = \cos\left(x - \frac{\pi}{2}\right)$
Period: 2π



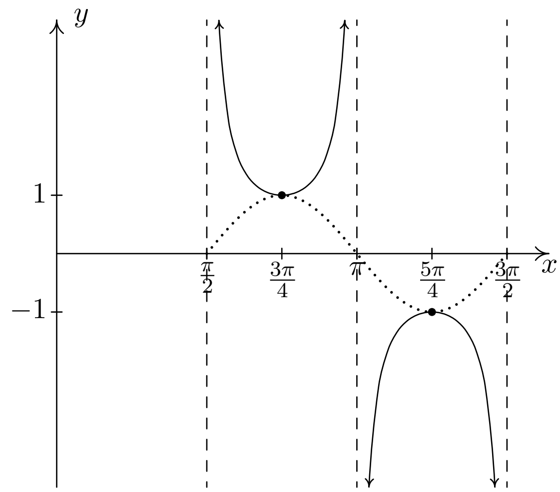
17. $y = -\csc\left(x + \dfrac{\pi}{3}\right)$
Start with $y = -\sin\left(x + \dfrac{\pi}{3}\right)$
Period: 2π



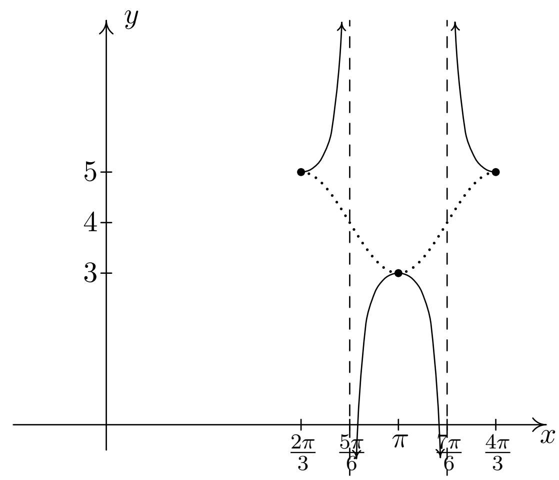
18. $y = -\dfrac{1}{3}\sec\left(\dfrac{1}{2}x + \dfrac{\pi}{3}\right)$
Start with $y = -\dfrac{1}{3}\cos\left(\dfrac{1}{2}x + \dfrac{\pi}{3}\right)$
Period: 4π



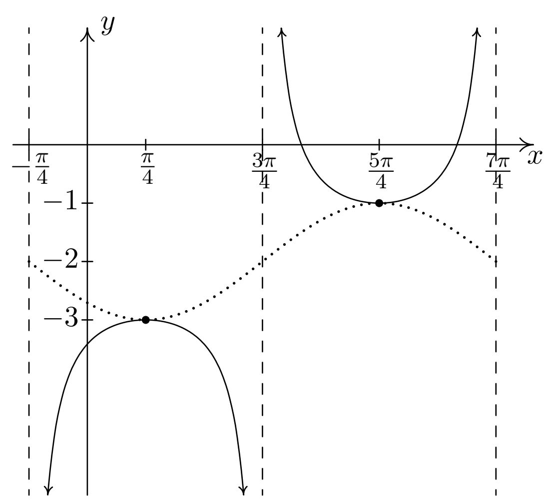
19. $y = \csc(2x - \pi)$
    Start with $y = \sin(2x - \pi)$
    Period: π



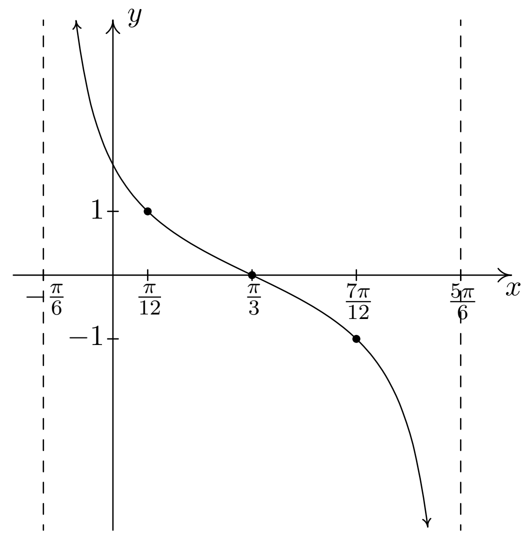
20. $y = \sec(3x - 2\pi) + 4$
    Start with $y = \cos(3x - 2\pi) + 4$
    Period: $\dfrac{2\pi}{3}$



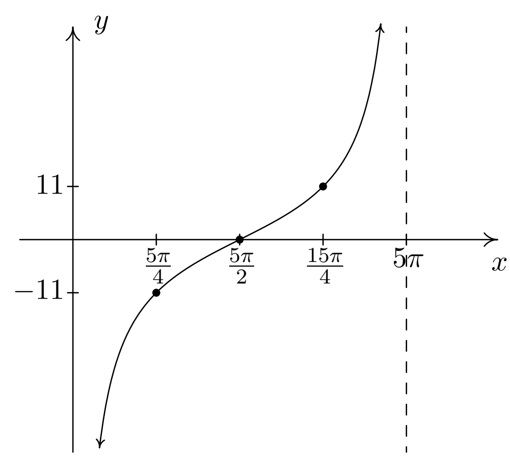
21. $y = \csc\left(-x - \dfrac{\pi}{4}\right) - 2$
    Start with $y = \sin\left(-x - \dfrac{\pi}{4}\right) - 2$
    Period: 2π



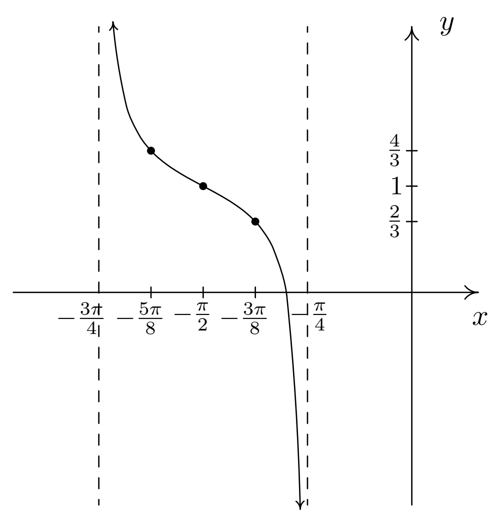
22. $y = \cot\left(x + \frac{\pi}{6}\right)$
Period: π



23. $y = -11\cot\left(\frac{1}{5}x\right)$
Period: 5π



24. $y = \frac{1}{3}\cot\left(2x + \frac{3\pi}{2}\right) + 1$
Period: $\frac{\pi}{2}$



25. $f(x) = \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1 = 2\sin\left(x + \frac{\pi}{4}\right) + 1 = 2\cos\left(x + \frac{7\pi}{4}\right) + 1$

26. $f(x) = 3\sqrt{3}\sin(3x) - 3\cos(3x) = 6\sin\left(3x + \frac{11\pi}{6}\right) = 6\cos\left(3x + \frac{4\pi}{3}\right)$

27. $f(x) = -\sin(x) + \cos(x) - 2 = \sqrt{2}\sin\left(x + \frac{3\pi}{4}\right) - 2 = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) - 2$

28. $f(x) = -\frac{1}{2}\sin(2x) - \frac{\sqrt{3}}{2}\cos(2x) = \sin\left(2x + \frac{4\pi}{3}\right) = \cos\left(2x + \frac{5\pi}{6}\right)$

29. $f(x) = 2\sqrt{3}\cos(x) - 2\sin(x) = 4\sin\left(x + \frac{2\pi}{3}\right) = 4\cos\left(x + \frac{\pi}{6}\right)$

30. $f(x) = \frac{3}{2}\cos(2x) - \frac{3\sqrt{3}}{2}\sin(2x) + 6 = 3\sin\left(2x + \frac{5\pi}{6}\right) + 6 = 3\cos\left(2x + \frac{\pi}{3}\right) + 6$

31. $f(x) = -\frac{1}{2}\cos(5x) - \frac{\sqrt{3}}{2}\sin(5x) = \sin\left(5x + \frac{7\pi}{6}\right) = \cos\left(5x + \frac{2\pi}{3}\right)$

32. $f(x) = -6\sqrt{3}\cos(3x) - 6\sin(3x) - 3 = 12\sin\left(3x + \frac{4\pi}{3}\right) - 3 = 12\cos\left(3x + \frac{5\pi}{6}\right) - 3$

33. $f(x) = \frac{5\sqrt{2}}{2}\sin(x) - \frac{5\sqrt{2}}{2}\cos(x) = 5\sin\left(x + \frac{7\pi}{4}\right) = 5\cos\left(x + \frac{5\pi}{4}\right)$

34. $f(x) = 3\sin\left(\frac{x}{6}\right) - 3\sqrt{3}\cos\left(\frac{x}{6}\right) = 6\sin\left(\frac{x}{6} + \frac{5\pi}{3}\right) = 6\cos\left(\frac{x}{6} + \frac{7\pi}{6}\right)$